Stability of the circular Couette flow of a ferrofluid in an axial magnetic field: Influence of polydispersity

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The gap between two concentric rotating cylinders is filled with a ferrofluid. A homogeneous magnetic field is applied parallel to the cylinder axis. The stability of the circular Couette flow is analyzed with different models that take into account the polydispersity of the ferrofluid to a varying degree. Their results are compared and their merits are discussed.

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I. INTRODUCTION

One of the many fascinating features of ferrofluids is the prospect of influencing the macroscopic flow by a magnetic field and vice versa [1–5]. One famous effect of this interaction is the dependence of the rotational viscosity of a ferrofluid on a magnetic field, the so-called magnetoviscous effect [6–9]. Quantitative investigations of the magnetoviscous effect are very important for technical applications. One method to quantify the rotational viscosity is the measurement of the critical angular velocity in the Taylor-Couette system [10–12].

The Taylor-Couette system has been the subject of research activities for many decades [13–20]. It consists of two concentric cylinders with radii r_1 and $r_2 > r_1$ which one can independently rotate with rotation rates $\mathbf{\Omega}_1 = \Omega_1 \mathbf{e}_z$ and $\mathbf{\Omega}_2 = \Omega_2 \mathbf{e}_z$. The gap between the cylinders is filled with a fluid. For a fixed value of Ω_2, there's a critical angular velocity Ω_1 of the inner cylinder at which the circular Couette flow (CCF) in the fluid becomes unstable. This critical rotation depends very sensitively on the viscous properties of the fluid. Figure 1 shows a schematic sketch of the system. Here we assume the system to be infinitely long and take therefore periodic boundary conditions in axial direction.

First stability analysises of CCF of a ferrofluid in a magnetic field use a stationary linearized magnetization equation to eliminate the magnetization [21–24]. Some recent works of Singh and Bajaj consider perturbations of the magnetization and the magnetic field [25,26]. Both, theoretical analyses as well as experimental investigations [10–12] show that magnetic fields stabilize the CCF. Further experiments are under way [27].

Real ferrofluids contain magnetic particles of different sizes [28,29]. This polydispersity can strongly influence the macroscopic magnetic properties of the ferrofluid [30,31]. So, recent experimental results for the magnetization of a rotating ferrofluid could only be reproduced with a model that take into account the different magnetic relaxation times of the different species of particles [31,32].

We investigate here the effect of polydispersity on the linear stability of the CCF in an homogeneous magnetic field $\mathbf{H}_{\text{ext}} = H_{\text{ext}} \mathbf{e}_z$ parallel to the cylinder axis. To that end we compare the results of a simple and a polydisperse Debye model. We use a stationary linearized approximation analogous to Niklas *et al.* [22,24] but also time dependent magnetization equations. Our investigations aim at two points: First, it is interesting to know, in which situations one needs a polydisperse model and in which situations it suffices to consider only the averaged properties of the ferrofluid. Secondly, it is important for comparisons with experimental data to quantify the influence of polydispersity.

II. EQUATIONS

The mass balance and the momentum balance yield equations for the flow field $\mathbf{u}$ and the pressure p in a fluid. For an incompressible ferrofluid with viscosity $\tilde{\eta}$, mass density ρ and kinematic viscosity $\nu = \tilde{\eta}/\rho$ these equations read

$$0 = \nabla \cdot \mathbf{u}, \tag{2.1}$$

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla^2 \mathbf{u} - \nabla p + 2(\mathbf{M} \cdot \nabla)\mathbf{H} + \nabla \times (\mathbf{M} \times \mathbf{H}). \tag{2.2}$$

Here the lengths are scaled with the gap width $d = r_2 - r_1$, times with the diffusion time d^2/ν, velocities with ν/d and the magnetic field $\mathbf{H}$ and the magnetization $\mathbf{M}$ with

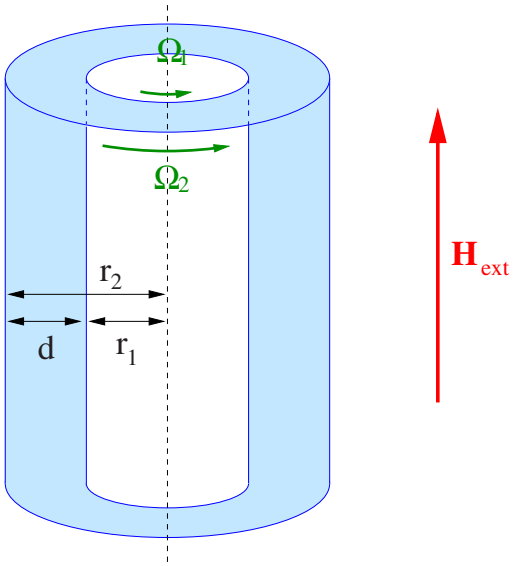

FIG. 1. (Color online) Schematic sketch of the Taylor-Couette system in an axial magnetic field.

$\sqrt{2\rho/\mu_0}\nu/d$ [25]. Furthermore $\mathbf{H}$ and $\mathbf{M}$ have to fullfill the Maxwell equations. Here we consider the magnetostatic case

$$\nabla \times \mathbf{H} = \mathbf{0}, \quad \nabla \cdot (\mathbf{H} + \mathbf{M}) = 0. \tag{2.3}$$

Additionally one needs an equation describing the magnetization dynamics. Shliomis deduced in his famous work [9] such a magnetization equation, denoted here as "S72"

$$\partial_t \mathbf{M} + (\mathbf{u} \cdot \nabla)\mathbf{M} = -\frac{1}{\tau}(\mathbf{M} - \mathbf{M}^{\mathrm{eq}}) + \boldsymbol{\Omega} \times \mathbf{M} + \kappa(\mathbf{M} \times \mathbf{H}) \times \mathbf{M} \tag{2.4}$$

with $\boldsymbol{\Omega} = \frac{1}{2}\nabla \times \mathbf{u}$. This model contains a single magnetic relaxation time τ and a constant $\kappa = (6\Phi\tilde{\eta})^{-1}$, where Φ is the volume fraction of the magnetic material. We furthermore use a simple Debye model (denoted as "DEBYE"), in which the nonlinear term $\kappa(\mathbf{M} \times \mathbf{H}) \times \mathbf{M}$ in S72 does not appear

$$\partial_t \mathbf{M} + (\mathbf{u} \cdot \nabla)\mathbf{M} = -\frac{1}{\tau}(\mathbf{M} - \mathbf{M}^{\mathrm{eq}}) + \boldsymbol{\Omega} \times \mathbf{M}. \tag{2.5}$$

Niklas *et al.* [22,24] considered stationary magnetizations $[\partial_t \mathbf{M} + (\mathbf{u} \cdot \nabla)\mathbf{M} = 0]$ near equilibrium with $|\mathbf{M} - \mathbf{M}^{\mathrm{eq}}|$ being small, which appear at not too high rotation rates ($\Omega\tau \ll 1$). In this case Eq. (2.4) [Eq. (2.5)] can be simplified to

$$\mathbf{M} - \mathbf{M}^{\mathrm{eq}} = c_N \boldsymbol{\Omega} \times \mathbf{H}, \quad c_N = \frac{\chi\tau}{1 + \kappa\tau\chi H^2}. \tag{2.6}$$

This approximation is denoted here as "NIKLAS(S72)" for $\kappa \neq 0$ and "NIKLAS(DEBYE)" for $\kappa = 0$.

In order to investigate the influence of polydispersity, we compare the results of DEBYE, NIKLAS(DEBYE), and NIKLAS(S72) with the results of a polydisperse model. This model (denoted as "POLY") considers a polydisperse ferrofluid as a mixture of ideal monodisperse paramagnetic fluids [30,31]. Then the resulting magnetization is given by $\mathbf{M} = \Sigma \mathbf{M}_j$, where $\mathbf{M}_j$ denotes the magnetization of the particles with diameter D_j. We assume that each $\mathbf{M}_j$ obeys a simple Debye relaxation dynamics described by

$$\partial_t \mathbf{M}_j + (\mathbf{u} \cdot \nabla)\mathbf{M}_j = -\frac{1}{\tau_j}(\mathbf{M}_j - \mathbf{M}_j^{\mathrm{eq}}) + \boldsymbol{\Omega} \times \mathbf{M}_j. \tag{2.7}$$

We take the equilibrium magnetization to be given by a Langevin function

$$\mathbf{M}_j^{\mathrm{eq}}(\mathbf{H}) = \chi_j(H)\mathbf{H} = w_j \mathcal{L}\left(\frac{\mu_0 \pi M_{\mathrm{mat}}}{6k_B T} D_j^3 H\right)\frac{\mathbf{H}}{H} \tag{2.8}$$

with the saturation magnetization of the material M_{mat} and the magnetization contribution $w_j(D_j)$. As relaxation rate we combine Brownian and Néel relaxation $\frac{1}{\tau_j} = \frac{1}{\tau_B^j} + \frac{1}{\tau_N^j}$. The relaxation times depend on the particle size by $\tau_B^j = \frac{\pi\tilde{\eta}}{2k_B T}(D_j + 2s)^3$ and $\tau_N^j = f_0^{-1} \exp(\frac{\pi K D_j^3}{6k_B T})$ with s the thickness of the nonmagnetic particle layer, and K the anisotropy constant. As an aside we mention that the models DEBYE and POLY coincide exactly for an ideal monodisperse ferrofluid.

In an analogous way to the approximation of Niklas *et al.* [22,24] one gets by adding the linearized stationary parts of the equations (2.7)

$$\mathbf{M} - \mathbf{M}^{\mathrm{eq}} = c_N \boldsymbol{\Omega} \times \mathbf{H}, \quad c_N = \sum_j \chi_j \tau_j \tag{2.9}$$

which is denoted here as "NIKLAS(POLY)." In this case the effect of polydispersity enters via one parameter, c_N, only.

By means of Eq. (2.6) [Eq. (2.9)] the magnetization can be eliminated in Eq. (2.2):

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla^2 \mathbf{u} - \nabla p_M + (\nabla c_N) \times [\mathbf{F} \times \mathbf{H}] + c_N\{\mathbf{F}(\nabla \cdot \mathbf{H}) - \mathbf{H}[\nabla \cdot \mathbf{F}] - \mathbf{H} \times [\nabla \times \mathbf{F}]\} \tag{2.10}$$

with $\mathbf{F} = \boldsymbol{\Omega} \times \mathbf{H}$. The index M of the pressure means that magnetic terms which can be written as a gradient are included in the pressure.

III. LINEAR STABILITY OF CCF

A. Basic state

The velocity field of CCF is given by

$$\mathbf{u}_{\mathrm{CCF}} = v_{\mathrm{CCF}} \mathbf{e}_\varphi, \quad v_{\mathrm{CCF}} = A_{\mathrm{CCF}} r + B_{\mathrm{CCF}} r^{-1}. \tag{3.1}$$

The constant factors A_{CCF} and B_{CCF} are determined by the no-slip boundary conditions $\mathbf{u}(r_i) = R_i \mathbf{e}_\varphi$ to be

$$A_{\mathrm{CCF}} = \frac{R_2 - \eta R_1}{1 + \eta}, \quad B_{\mathrm{CCF}} = \frac{\eta}{1 - \eta}\frac{R_1 - \eta R_2}{1 - \eta^2} \tag{3.2}$$

with the radius ratio $\eta = r_1/r_2$ and the Reynolds numbers $R_i = \Omega_i r_i d/\nu$. The external applied field $\mathbf{H}_{\mathrm{ext}} = H_{\mathrm{ext}} \mathbf{e}_z$ in axial direction leads in the simplest case to an internal field and a magnetization

$$\mathbf{H}^{\mathrm{CCF}} = \mathbf{H}_{\mathrm{ext}}, \quad \mathbf{M}^{\mathrm{CCF}} = \chi(H_{\mathrm{ext}})\mathbf{H}_{\mathrm{ext}}, \quad \mathbf{M}_j^{\mathrm{CCF}} = \chi_j(H_{\mathrm{ext}})\mathbf{H}_{\mathrm{ext}}. \tag{3.3}$$

B. Perturbations of CCF

As a first approximation we take the simplified magnetization equation (2.9) and neglect perturbations of the magnetic field. In this case small perturbations $\mathbf{u} = u\mathbf{e}_r + v\mathbf{e}_\varphi + w\mathbf{e}_z$ and p of CCF obey the linearized equations

$$\nabla \cdot \mathbf{u} = 0, \tag{3.4}$$

$$\partial_t u = -\frac{v_{\mathrm{CCF}}}{r}\partial_\varphi u + \frac{2v_{\mathrm{CCF}}}{r} v - \partial_r p + \frac{1}{r}\partial_\varphi\left[\frac{1}{r}\partial_\varphi u - \left(\partial_r + \frac{1}{r}\right)v\right] + (1 + S)\partial_z(\partial_z u - \partial_r w), \tag{3.5a}$$

$$\partial_t v = -2A_{\mathrm{CCF}} u - \frac{v_{\mathrm{CCF}}}{r}\partial_\varphi v - \frac{1}{r}\partial_\varphi p + \partial_r\left[\left(\partial_r + \frac{1}{r}\right)v - \frac{1}{r}\partial_\varphi u\right] + (1 + S)\partial_z\left(\partial_z v - \frac{1}{r}\partial_\varphi w\right), \tag{3.5b}$$

$$\partial_t w = -\frac{v_{\mathrm{CCF}}}{r}\partial_\varphi w - \partial_z p - (1+S)\left[\left(\partial_r + \frac{1}{r}\right)(\partial_z u - \partial_r w) + \frac{1}{r}\partial_\varphi\left(\partial_z v - \frac{1}{r}\partial_\varphi w\right)\right]. \quad (3.5c)$$

The magnetic properties of the system are included in the parameter $S=\frac{c_N}{2}H_{\mathrm{ext}}^2$.

In the second step, we consider perturbations $\mathbf{M}$ of the magnetization and $\mathbf{H}$ of the internal field, too. The equilibrium magnetization is assumed to be independent of perturbations $\mathbf{M}^{\mathrm{eq}}(\mathbf{H}^{\mathrm{CCF}}+\mathbf{H}) \simeq \mathbf{M}^{\mathrm{eq}}(\mathbf{H}^{\mathrm{CCF}})$. Then the perturbations of the internal field can be eliminated and we get the linearized equations

$$0 = \nabla \cdot \mathbf{u}, \quad (3.6)$$

$$\partial_t \mathbf{u} = -(\mathbf{u}\cdot\nabla)\mathbf{u}_{\mathrm{CCF}} - (\mathbf{u}_{\mathrm{CCF}}\cdot\nabla)\mathbf{u} + \nabla^2\mathbf{u} - \nabla p + [\nabla\times(\mathbf{M}\times\mathbf{H}_{\mathrm{ext}}) - \chi\mathbf{H}_{\mathrm{ext}}(\nabla\cdot\mathbf{M})], \quad (3.7)$$

$$\partial_t \mathbf{M}_j = -(\mathbf{u}_{\mathrm{CCF}}\cdot\nabla)\mathbf{M}_j + \frac{1}{2}(\nabla\times\mathbf{u}_{\mathrm{CCF}})\times\mathbf{M}_j - \frac{1}{\tau_j}\mathbf{M}_j + \frac{1}{2}\chi_j(\nabla\times\mathbf{u})\times\mathbf{H}_{\mathrm{ext}}. \quad (3.8)$$

C. Stability analysis

We use for the perturbations at the stability boundary the ansatz

$$(\mathbf{u},p,\mathbf{M})(\mathbf{r},t) = e^{-i\omega t}\sum_{m,k}(\mathbf{U},P,\hat{\mathbf{M}})(r,m,k)e^{im\varphi+ikz}. \quad (3.9)$$

Inserting this ansatz in Eqs. (3.4) and (3.5) [Eqs. (3.6)–(3.8)] yields the six linear ordinary differential equations

$$\partial_r U = -\frac{1}{r}U - \frac{im}{r}V - ikW, \quad (3.10a)$$

$$\partial_r V = \frac{im}{r}U - \frac{1}{r}V + Y, \quad (3.10b)$$

$$\partial_r W = ikU + (1+S)^{-1}Z + (1+\chi)H_{\mathrm{ext}}\hat{M}_r, \quad (3.10c)$$

$$\partial_r P = -\left(-i\omega + v_{\mathrm{CCF}}\frac{im}{r}\right)U + \frac{2v_{\mathrm{CCF}}}{r}V - \frac{im}{r}Y - ikZ - ik\chi H_{\mathrm{ext}}\hat{M}_r, \quad (3.10d)$$

$$\partial_r Y = 2A_{\mathrm{CCF}}U + \left(-i\omega + v_{\mathrm{CCF}}\frac{im}{r}\right)V + \frac{im}{r}P - ikX + ik\chi H_{\mathrm{ext}}\hat{M}_\varphi, \quad (3.10e)$$

$$\partial_r Z = \left(-i\omega + v_{\mathrm{CCF}}\frac{im}{r}\right)W + ikP - \frac{1}{r}Z + \frac{im}{r}X \quad (3.10f)$$

with the abbreviation

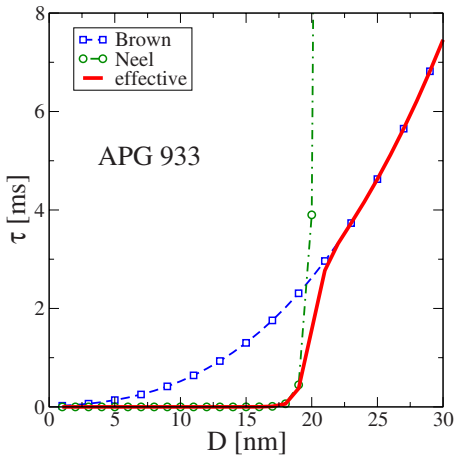

FIG. 2. (Color online) Brown, Néel, and effective relaxation times of the ferrofluid APG933.

$$X = (1+S)\left(ikV - \frac{im}{r}W\right) + (1+\chi)H_{\mathrm{ext}}\hat{M}_\varphi. \quad (3.11)$$

When we use NIKLAS(DEBYE), NIKLAS(POLY), or NIKLAS(S72), we set $S=\frac{c_N}{2}H_{\mathrm{ext}}^2$ and $\hat{\mathbf{M}}=0$. For DEBYE and POLY we take $S=0$ and calculate $\hat{\mathbf{M}}=\Sigma_j\hat{\mathbf{M}}_j$ from the linear algebraic equations

$$\left(-i\omega + \frac{1}{\tau_j} + v_{\mathrm{CCF}}\frac{im}{r}\right)\hat{M}_r^j = +B_{\mathrm{CCF}}r^{-2}\hat{M}_\varphi^j - \frac{1}{2}\chi^j H_{\mathrm{ext}}Z - \frac{1}{2}\chi^j(1+\chi)H_{\mathrm{ext}}^2\hat{M}_r, \quad (3.12a)$$

$$\left(-i\omega + \frac{1}{\tau_j} + v_{\mathrm{CCF}}\frac{im}{r}\right)\hat{M}_\varphi^j = -B_{\mathrm{CCF}}r^{-2}M_r^j + \frac{1}{2}\chi^j H_{\mathrm{ext}}X - \frac{1}{2}\chi^j(1+\chi)H_{\mathrm{ext}}^2\hat{M}_\varphi, \quad (3.12b)$$

$$\left(-i\omega + \frac{1}{\tau_j} + v_{\mathrm{CCF}}\frac{im}{r}\right)\hat{M}_z^j = 0 \leftrightarrow \hat{M}_z^j = 0. \quad (3.12c)$$

It remains to solve the six linear differential equations (3.10) with the six no-slip boundary conditions at the cylinders ($r=r_1$ and $r=r_2$) [37]

$$\mathbf{U}(r=r_i) = 0. \quad (3.13)$$

To that end we use a shooting method [33,34]. At fixed values of R_2, η, m, k, and H_{ext} the marginal values R_1^{marg} and ω_{marg} are calculated. The critical values R_{1c}, ω_c, and k_c are given by the minimum of the marginal curve $R_1^{\mathrm{marg}}(k)$.

IV. RESULTS

For the numerical calculations, we take typical values for the ferrofluid APG933 of FerroTec [29–32]: $M_{\mathrm{mat}} \simeq 450$ kA/m, $\Phi=4.1\%$, $\tilde{\eta}=0.5$ Pa s, $s=2$ nm, $f_0=10^9$ Hz

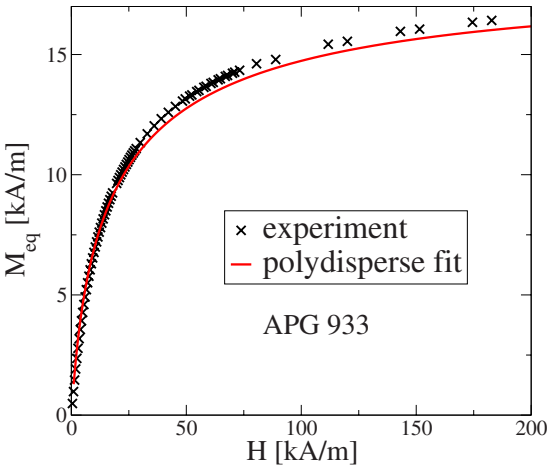

FIG. 3. (Color online) Equilibrium magnetization of the ferrofluid APG933: experimental data (×) and fit with a lognormal distribution.

and $K=15\ \mathrm{kJ/m^3}$. One can find in Fig. 2 the effective relaxation times calculated with these parameters. We furthermore use as input the experimental equilibrium magnetization $M_{\mathrm{eq}}(H)$ of APG933 shown in Fig. 3 and the magnetic weights of Fig. 4 obtained by fitting to a lognormal distribution. As the single relaxation time in (2.4) [Eq. (2.5)] we take a value of 0.5×10^{-3} s which is comparable in size with the averaged effective relaxation times as well as experimental results [35,36].

Figure 5 shows the magnetic field parameter S as a function of H_{ext}, calculated for the three models NIKLAS(DEBYE), NIKLAS(POLY), and NIKLAS(S72). In the approximation used by Niklas *et al.*, the magnetic field and all magnetic properties of the ferrofluid are only represented by the magnetic field parameter S in the basic equations. Thus, all critical values only depend on this parameter. With increasing S, we found R_{1c} and ω_c to also increase while k_c decreases as shown in Fig. 6.

According to the used model, the magnetic field parameter S depends on H_{ext} in different ways. Therefore, the three models yield different critical values (see Figs. 7 and 8).

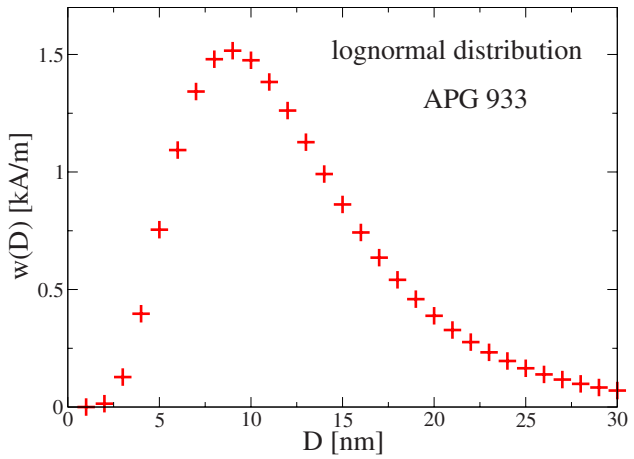

FIG. 4. (Color online) Lognormal distribution of the ferrofluid APG933.

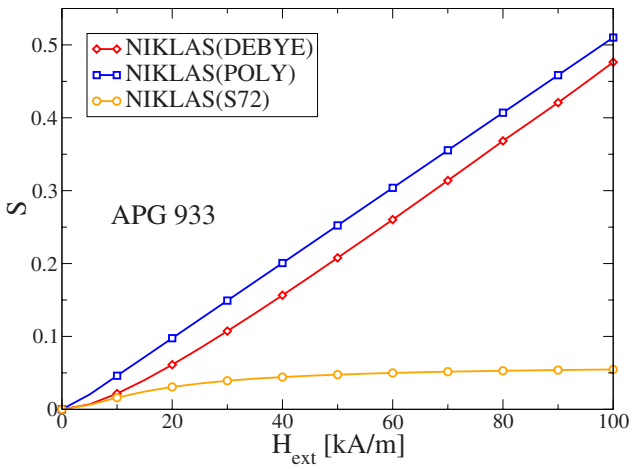

FIG. 5. (Color online) Magnetic field parameter S as a function of H_{ext} for the ferrofluid APG933.

NIKLAS(POLY) causes the largest, NIKLAS(S72) the smallest change of the critical values compared to the case without a magnetic field. Finally, results by NIKLAS(DEBYE) differ only marginally from those of NIKLAS(POLY).

The critical onsets calculated with the full relaxation equations (DEBYE and POLY) lie between the results of NIKLAS(POLY) and NIKLAS(S72). This does not hold for the critical wave numbers k_c and small H_{ext}. Namely, the polydisperse relaxation equations lead to increasing values of k_c at small H_{ext} and $m=1$ (Fig. 8). For a corotating ($R_2>0$) or counter-rotating ($R_2<0$) outer cylinder, one can find similar results (see Figs. 9–12).

Considering the full polydisperse relaxation equations (POLY), the polydispersity reduces the effect of the magnetic field. In the approximation of Niklas *et al.*, the polydisperse results are very similar to the results of DEBYE with one

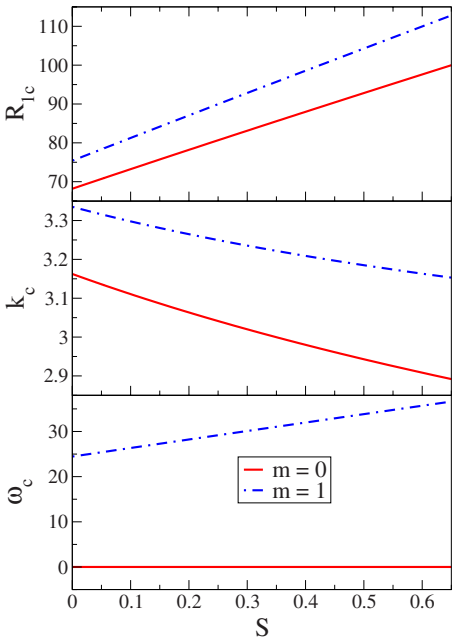

FIG. 6. (Color online) Critical values R_{1c}, ω_c, and k_c as a function of the magnetic field parameter S for $\eta=0.5$ and $R_2=0$.

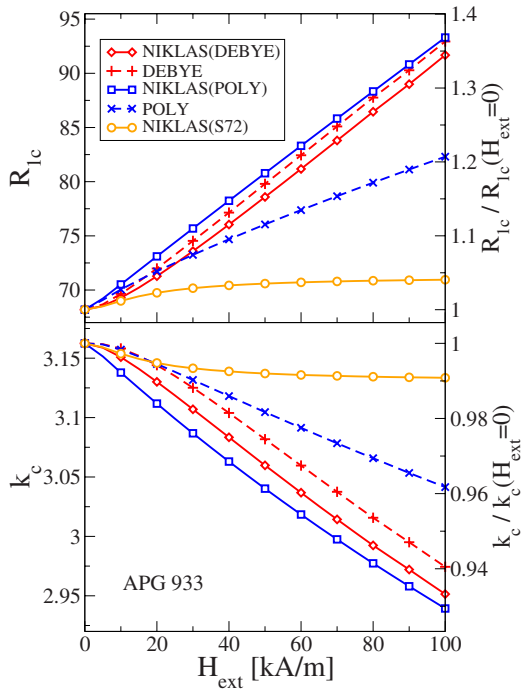

FIG. 7. (Color online) Critical values R_{1c} and k_c as a function of H_{ext} for $m=0$ ($\omega_c=0$), $\eta=0.5$, and $R_2=0$.

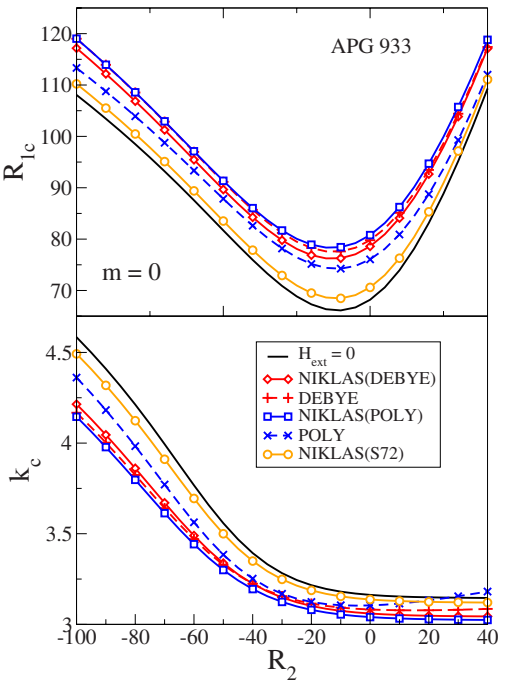

FIG. 9. (Color online) Critical values R_{1c} and k_c as a function of R_2 for $m=0$ ($\omega_c=0$), $\eta=0.5$, and $H_{\text{ext}}=50$ kA/m.

relaxation time. But the difference between DEBYE and POLY is smaller than the difference between NIKLAS(POLY) and NIKLAS(S72). It should be possible to obtain the polydisperse stability boundary $R_{1c}(H_{\text{ext}})$ with the Niklas approximation and an appropriately chosen relation between S and H_{ext}. The main difference between POLY and the other models is the critical wave number k_c in the case of corotation. k_c increases in a wide range of magnetic field amplitudes if one uses POLY. In contrast to that, this behavior changes using the other models (Figs. 11 and 12).

V. SUMMARY AND CONCLUSION

We investigated five magnetization models, namely the simple Debye model (DEBYE), the polydisperse Debye

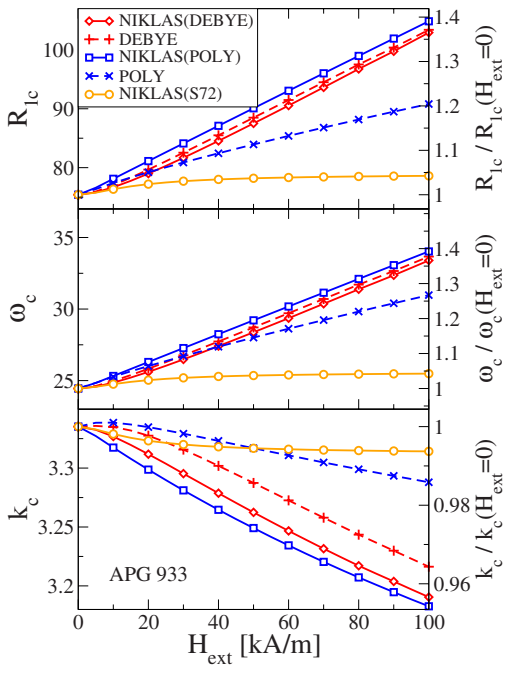

FIG. 8. (Color online) Critical values R_{1c}, ω_c, and k_c as a function of H_{ext} for $m=1$, $\eta=0.5$, and $R_2=0$.

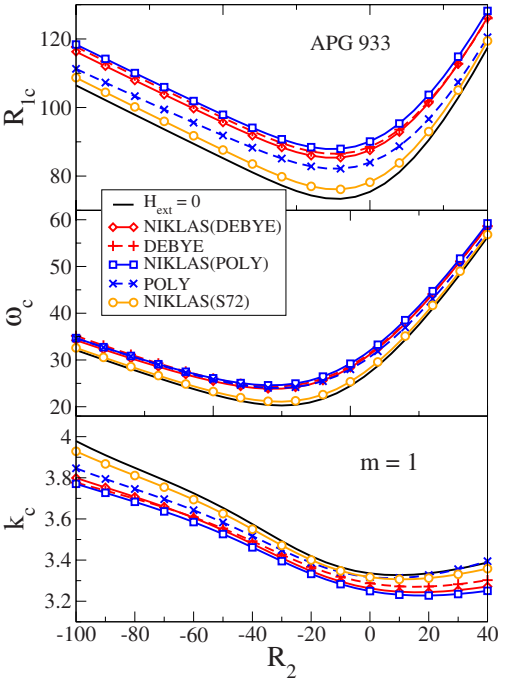

FIG. 10. (Color online) Critical values R_{1c}, ω_c, and k_c as a function of R_2 for $m=1$, $\eta=0.5$, and $H_{\text{ext}}=50$ kA/m.

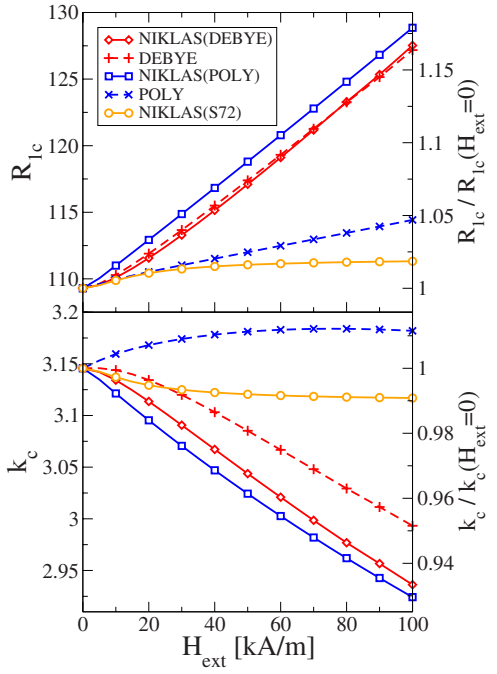

FIG. 11. (Color online) Critical values R_{1c} and k_c as a function of H_{ext} for $R_2=40$ (corotating cylinders), $\eta=0.5$, and $m=0$ ($\omega_c=0$).

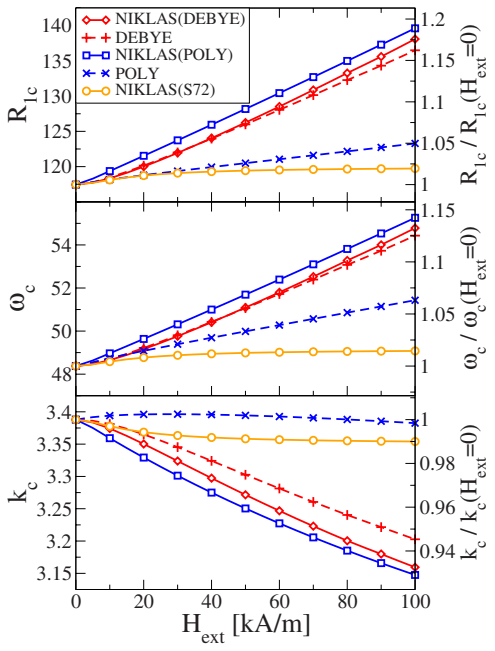

FIG. 12. (Color online) Critical values R_{1c}, ω_c, and k_c as a function of H_{ext} for $R_2=40$ (corotating cylinders), $\eta=0.5$, and $m=1$.

model (POLY), and their respective Niklas approximations NIKLAS(DEBYE) and NIKLAS(POLY) as well as the Niklas approximation NIKLAS(S72) of the Shliomis model S72. All Niklas-approximations differ only in their respective dependence of the magnetic field parameter S on the external magnetic field H_{ext}.

Generally, magnetic fields stabilize the CCF against $m=0$ and $m=1$ disturbances. Depending on the used model, the strength of this stabilization differs: In contrast to DEBYE, NIKLAS(DEBYE), and NIKLAS(POLY) with a stronger and roughly similar stabilization, POLY and NIKLAS(S72) on the other hand lead to a significant weaker stabilizing effect.

For a given axial magnetic field, we found the influence of the polydispersity on the linear stability threshold for CCF to be smaller than the effect of a nonvanishing value for κ in NIKLAS(S72). It should be possible to obtain the stability threshold $R_{1c}(H_{\text{ext}})$ for POLY by applying the Niklas approximation and an appropriately chosen relation between S and H_{ext}. Such a relation could also be obtained by fitting experimental results. It remains to be seen how far the accuracy of experiments is sufficient to discriminate between all of the different theoretical models.

ACKNOWLEDGMENT

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